On some Difficulties connected with the Determination of the Diameter of Mars. By E. J. Stone, Esq.

Our determinations of the diameters of the planets are complicated by the existence of systematic personal errors in the observers and, to some extent, by constant errors depending upon the instrument used. But the determinations of the diameter of *Mars* which have been made from time to time, have differed so much amongst themselves that there are grounds for suspecting the existence of some source of systematic error which has been overlooked.

Thus, in the *Monthly Notices* for November, 1880, Mr. Downing has inferred, from a discussion of all the measures of the vertical diameter of *Mars* which have been made with the Greenwich Transit Circle, that Le Verrier's value of the angular diameter of *Mars*, at the Sun's mean distance, 11″·10, requires to be diminished to 9″·70. The results deduced from two extensive series of observations, with many changes amongst the observers, and with different adopted values of the diameter, are closely accordant; and the result has been correctly deduced from the observations on the assumptions made. But Le Verrier's result equally represents the observations on the assumption which he made.

The observations at Greenwich are made by a large number of persons; and, although some errors, of an unsystematic character, do thus creep into the work, and the discussion of the results is greatly complicated, the facilities afforded for the examination of questions connected with personal errors are very great.

I have collected, from the Greenwich volumes, the measures of the vertical diameters of *Mars* made 1851 to 1878, and assuming that the true value of the angular diameter =Tab.$(1+y)$, the personal errors of the observers are given on two suppositions.

Table I.

Personal Equations.

With Le Verrier's Diameter 11″·10 under the heading V.

With Downing's Diameter 9″·697 under the heading D.

Period 1851-1865.

Observer's Name.	V.	D.	Factor of y.	Number of Observations.
Breen, J.	−0″·29	+1″·35	−10″·40	1
Breen, H.	+0·23	+2·28	−13·00	2
Bowden	+0·42	+2·53	−13·40	1
Carpenter, J.	−1·34	+0·74	−13·18	15
Criswick	−0·83	+1·54	−15·01	31

Observer's Name.	V.	D.	Factor of y.	Number of Observations.
Christy, W. J.	$-1''.46$	$+0''.14$	$-10''.01$	2
Chappell	$+1.23$	$+3.87$	-16.71	11
Davis	-1.84	$+1.35$	-20.20	4
Dolman	-1.25	$+2.14$	-21.45	2
Dunkin	-0.66	$+1.39$	-13.01	40
Eaton	$+0.85$	$+3.92$	-19.37	6
Ellis, W.	$+1.09$	$+3.39$	-14.60	54
Henry	$+0.28$	$+2.17$	-11.88	15
Henderson	$+0.30$	$+2.06$	-11.10	11
Kerschner	-0.86	$+1.89$	-17.43	12
Lynn	$+1.40$	$+3.60$	-13.92	12
Main	$+0.80$	$+2.88$	-13.20	1
Newcomb	$+5.14$	$+7.03$	-12.00	1
Nash	-0.89	$+1.50$	-15.33	3
Roberts	-1.11	$+1.48$	-16.38	8
Rogerson	$+1.19$	$+3.15$	-12.36	5
Stone	$+1.73$	$+4.08$	-14.85	6
Taylor, H.	$+0.02$	$+1.90$	-11.85	4
Taylor, F.	$+0.31$	$+2.39$	-13.20	1
Talmage	$+3.76$	$+4.55$	-5.00	1
Wakelin	-1.62	$+0.81$	-15.40	4
Weighted Means	$+0.06$	$+2.32$	-14.32	253 = Sum

These coefficients of y require to be increased in the proportion of $\frac{11.10}{8.87}$ or 1.25 to be strictly comparable with those of Table II. This correction has been allowed for in the work.

Table II.

Personal Equations.

With Le Verrier's Diameter $11''.10$ under the heading V.

With Downing's Diameter $9''.697$ under the heading D.

Period 1866–1878.

Observer's Name.	V.	D.	Factor of y.	Number of Observations.
Baker	$+2''.01$	$+3''.73$	$-13''.65$	4
Bromley	-1.26	$+2.19$	-27.60	4
Christie, W. H. M.	-1.74	-0.04	-13.52	5
Criswick	-0.37	$+1.78$	-15.77	40
Carpenter, J.	-0.79	$+0.95$	-13.89	20
Carpenter, H.	$+0.74$	$+2.67$	-15.25	8

Observer's Name.	V.	D.	Factor of y.	Number of Observations.
	″	″	′	
Dennison	−0·94	+1·25	−17·43	3
Dunkin	−0·70	+1·09	−14·17	6
Downing	−0·14	+2·13	−18·04	26
Ellis	+1·12	+3·01	−15·00	23
Graham	+2·62	+5·03	−19·14	7
Harding	+0·15	+2·62	−19·60	1
Jenkins	+0·71	+2·96	−17·94	7
Keating	+0·18	+2·84	−21·10	2
Kerschner	+1·83	+3·68	−14·65	4
Laird	+1·63	+3·56	−15·28	5
Lynn	+2·34	+4·19	−14·73	18
Pett	−1·14	+1·27	−19·10	3
Pead	−2·03	+1·23	−25·90	3
Plummer	+1·15	+2·85	−13·50	2
Potts	+1·11	+3·16	−16·28	6
Power	+0·12	+2·39	−18·18	6
Pulley	−0·49	+1·44	−15·26	7
Robinson	+0·04	+2·56	−20·20	7
Thackeray	+0·81	+3·35	−20·13	21
Wickham	+0·06	+2·73	−21·15	2
Wright	−0·02	+1·92	−15·40	1
Weighted Means	+0·33	+2·47	−16·74	241 = Sum

It will be seen that the measures of different observers may differ by nearly 3″ in cases where the number of observations would render any mere errors of observation small, and where the factors of the adopted diameter are too nearly alike to render any adjustment of these differences by a change of the adopted diameter possible. A comparison of the results of the two series will also show that these personal differences remain, for the principal observers, almost unchanged, and that the methods of observing contacts adopted by experienced observers sensibly and systematically differ, or that real personal equations to a very sensible amount do, as a matter of fact, exist in the measures of the diameter of *Mars*.

The results given in these tables appear to me of some practical importance: they afford a very necessary caution against the acceptance of any measured diameters as definitive when all the measures have been made by the same person, unless a sufficient range exists amongst the measured diameters to allow of an accurate separation of the constant and variable parts of the observed diameter.

If we adopt Le Verrier's value, the mean error of all the observed diameters in Table (1) are destroyed, and the mean error in Table (2) is very small. It is clear, therefore, that Le Verrier has assumed that, in the mean of a large number of measures made by many different observers, the personal errors will mutually destroy each other's effects. The legitimacy of this assumption would appear to be confirmed by the smallness of the mean residual errors in both series of the Greenwich observations, although very considerable changes in the observing staff have taken place. But, of course, Le Verrier's assumption would be untrue, and his result vitiated to a corresponding extent, if there be any systematic source of error which affects the measured diameters of *all the observers* in the same way. Such a source of constant error is presented by telescopic irradiation.

The formula adopted by Mr. Downing is one which has previously been frequently adopted in similar discussions, and distinctly contemplates the existence of *constant* errors in the measures. Mr. Downing assumes that

$$x + y \text{ (Tab. Diameter)} = \text{Obs. Diameter} - \text{Tab. Diameter},$$

and attempts to determine both x and y from the equations of condition. But, unless the observations are all made by the same person, or corrections are applied for personal errors, an assumption is virtually made that the measures of large and small diameters are equally distributed amongst the different observers. If this condition is satisfied, and the variations between the factors of y are sufficiently large to allow of an accurate separation of x and y, then the diameter thus determined has certainly greater claims to our acceptance than that found by putting $x=0$, which is really the form adopted by Le Verrier. I have separately discussed the observations made by the principal observers, who have observed diameters over a sufficient range; the results confirm the value of y obtained by Mr. Downing, and prove, as might be expected from the large number of observations employed, that the distribution of the measures has been sufficiently uniform to avoid any serious effects on the determination of y. But an inspection of Tables (1) and (2) shows that it is impossible to accept Mr. Downing's value of the diameter of *Mars* without accepting as a fact that the error of nearly 500 measures of the diameter of *Mars* made with an 8-inch telescope by 47 observers is more than $2''$.

It appears to me very difficult to accept such a conclusion. The irradiation constant for the planet *Mars*, with such a telescope, can hardly amount to one-fifth of the quantity under consideration; and, although the mean personal error of the observers is included in the quantity, it is difficult to conceive that the mean personal error of so many observers can amount to nearly as large a quantity as the whole range, of personal error, which is well established amongst the large number of observers whose measures are compared in the present

paper. It appears to me, therefore, that there must be some systematic error which vitiates the result by rendering untrue the assumption that all deviations of the measured diameters from the formula

$$x + y \text{ (Tab. Diameter)} = \text{Obs. Diam.} - \text{Tab. Diam.}$$

arise from mere chance errors of observation. If we represent the errors geometrically by points, whose abscissæ are the tabular diameters and ordinates the excess of observed diameters over the tabular diameters, we ought, on the suppositions which underlie our investigations, to be able to draw a straight line amongst these points, so that there should be no unequal preponderance of points on either side of the line throughout its length. I have traced the curves for several of the observers, and find that such is not the case. To test the point I have proceeded as follows:—

(1) All the values of excess of observed over tabular diameters have been reduced to an uniform standard with the assumed value of 11″·10 at the mean distance.
(2) Corrections for personal equations have been applied from the Tables (1) and (2).
(3) No observations have been employed, unless the same observer has made observations when the diameter was very small, near the mean, and large. This avoids any errors creeping into the work from imperfect correction for personal error.

The measures were then divided into groups with tabular diameters:—

5″–7″; 7″–9″; 9″–12″; 12″–15″; 15″–17″;
17″–19″; 19″–22″; 22″–25″; above 25″.

The following table gives the results:—

Mean Factor.	No. of Observations.	Assumed Diameter at Earth's Mean Distance = 9·71	= 10″·73	= 11″·10
″		″	″	″
7·48	5	+2·94	+2·24	+2·00
9·11	25	+2·39	+1·54	+1·24
12·21	49	+2·15	+1·01	+0·61
15·32	68	+2·05	+0·62	+0·12
17·80	80	+2·25	+0·60	+0·01
20·85	28	+1·76	−0·18	−0·87
23·21	35	+2·36	+0·21	−0·56
24·43	18	+2·15	−0·11	−0·92
27·33	11	+2·50	−0·04	−0·94

If these points be laid down to scale, the break of continuity in passing from the smaller measures is distinctly seen. It will be found that the straight line represented by Mr. Downing's solu-